DISPERSION THEORY OF SURFACE WAVES ON AN INHOMOGENEOUS PLASMA IN A STRONG HIGH-FREQUENCY FIELD

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The spectrum of the characteristic surface oscillations and stability of a plasma in a strong high-frequency (hf) electric field are studied. It is shown that inhomogeneity of the plasma leads to spatial dispersion and to specific damping of stable oscillations of a plasma in an external hf field, the frequency of which greatly exceeds the plasma frequency. A systematic theory of parametric resonance at the frequency of the electronic surface oscillations is developed taking account of the inhomogeneity of the plasma.

1. The dispersion theory of surface waves of a plasma in a strong high-frequency (hf) field [1] has been developed for the case of a homogeneous plasma with a sharp boundary. In actual experiments, the approximation of a homogeneous plasma with a sharp boundary is not always justified. It is known [2, 3] that inhomogeneity of the plasma has a significant influence on the spectrum of hf surface oscillations. Thus, in the case where the characteristic dimension of an inhomogeneity near the plasma boundary greatly exceeds the Debye radius, the spatial dispersion and damping coefficient for hf surface waves is completely defined by plasma inhomogeneity effects. From the theory of parametric resonance in an unbounded homogeneous plasma [4], it is known that for strong hf fields the occurrence of spatial dispersion of plasma waves substantially changes the picture of plasma instabilities. Below, it is shown that analogous effects also occur for parametric resonance at the frequency of hf surface waves in a bounded inhomogeneous plasma. It is established that an aperiodic instability arises not only at a frequency of the external field ω_0, lower than the frequency of the surface waves $\omega_p/(1+\varepsilon_0)^{1/2}$, but also for $\omega_0 > \omega_p/(1+\varepsilon_0)^{1/2}$. In addition, situations are possible in which the parametric instability is dissipative.

Besides studying the peculiarities of the parametric resonance, the present work also investigates the influence of plasma inhomogeneities on the spectrum of unstable surface oscillations in an external hf field.

2. We consider a plasma with density n(z) rapidly rising in a transition layer $0 \le z \le a$ and changing relatively slowly for $z > a$, so that the characteristic dimension of the inhomogeneity

$$L = (\partial \ln n(z) / \partial z|_{z=a})^{-1} \gg a .$$

For such conditions, in the plasma there exist weakly damped surface waves with wave vector $\mathbf{k}_\parallel$, directed along the plasma boundary and satisfying [2, 3]

$$a^{-1} \gg k_\parallel \gg L^{-1}. \tag{2.1}$$

We assume that the external electric field vector is oriented along the plasma boundary. For sufficiently large $k_\parallel$, when the region of field localization of the surface wave (equal to $1/k_\parallel$) is much less than the penetration depth of the external field $c/(\omega_p^2-\omega_0^2)^{1/2}$, the latter may be considered homogeneous: $\mathbf{E} = \mathbf{E}_0 \sin\omega_0 t$.

The dispersion relations for the lf (with frequency $\omega \ll \omega_0$) and the hf (with frequency $\omega \pm n\omega_0$) surface waves in a strong hf field have the form

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$$1 + \delta\varepsilon_i^{(0)}(a) \sum_{n=-\infty}^{\infty} \frac{J_n^2}{R^{(n)}} = 0 \tag{2.2}$$

$$R^{(n)} = [1 + \varepsilon_0 + \delta\varepsilon_e^{(n)}(a)] \Big\{ 1 - \frac{1}{2k_\parallel} \frac{\partial}{\partial z} \frac{\delta\varepsilon_i^{(0)}(z)}{\delta\varepsilon_i^{(0)}(a)} \Big|_{z=a} -$$

$$- k_\parallel \int_0^a dz \frac{\delta\varepsilon_i^{(0)}(z)}{\delta\varepsilon_i^{(0)}(a)} - k_\parallel \varepsilon_0 \sum_{m=-\infty}^{\infty} \frac{J_m^2}{1 + \varepsilon_0 + \delta\varepsilon_e^{(m)}(a)} \int_0^a dz \frac{1 + \delta\varepsilon_e^{(n)}(a)}{1 + \delta\varepsilon_e^{(m)}(z)} \times$$

$$\times \left[\frac{\delta\varepsilon_i^{(0)}(z)}{1 + \delta\varepsilon_e^{(n)}(z)} - \frac{\delta\varepsilon_i^{(0)}(a)}{1 + \delta\varepsilon_e^{(n)}(a)} \right] \left[1 + \delta\varepsilon_i^{(0)}(z) \sum_{s=-\infty}^{\infty} \frac{J_s^2}{1 + \delta\varepsilon_e^{(s)}(z)} \right]^{-1} \Big\} +$$

$$+ \frac{1}{2k_\parallel} \frac{\partial}{\partial z} [1 + \delta\varepsilon_e^{(n)}(z)]_{z=a} + k_\parallel \varepsilon_0 \int_0^a dz \frac{1 + \delta\varepsilon_e^{(n)}(a)}{1 + \delta\varepsilon_e^{(n)}(z)} +$$

$$+ k_\parallel \int_0^a dz [1 + \delta\varepsilon_e^{(n)}(z)] + \frac{(1 + \varepsilon_0)(\omega + n\omega_0)^2}{2k_\parallel^2 c^2}$$

$$\delta\varepsilon_\alpha^{(n)}(z) = -\omega_{L\alpha}^2(z) / (\omega + n\omega_0)^2, \quad \mathbf{r}_E = e\mathbf{E}_0 / m_e\omega_0^2.$$

Here ε_0 is the dielectric permeability of the medium surrounding the plasma: $\delta\varepsilon_\alpha^{(n)}(z)$ is the contribution of particles of type α in the linear dielectric permeability of a cold plasma; J_n is the Bessel function of argument $\mathbf{k}_\parallel \mathbf{r}_E$; and $\mathbf{r}_E$ is the amplitude of electron oscillations in the hf field.

3. Using the dispersion relation (2.2), we first examine the spectra of the weakly damped surface oscillations in the case of external fields with frequencies much greater than the plasma frequency, $\omega_p = [\omega_{Le}^2(a) + \omega_{Li}^2(a)]^{1/2}$. In this case there may exist both hf and lf surface waves of frequency Ω and damping coefficient γ' given by:

$$\Omega^2 = \omega_*^2 \Big\{ 1 - \frac{\omega_*^2}{4k_\parallel^2 c^2} + \frac{k_\parallel}{1 + \varepsilon_0} \int_0^a \frac{dz}{\varepsilon(\omega_*, z)} [\varepsilon_0^2 - \varepsilon^2(\omega_*, z)] + \frac{1}{2k_\parallel} \frac{\partial \ln\varepsilon(\omega_*, z)}{\partial z} \Big|_{z=a} \Big\} \tag{3.1}$$

$$\gamma' = \frac{\pi\varepsilon_0^2 \omega_* k_\parallel}{2(1 + \varepsilon_0)} \int_0^a dz \delta[\varepsilon(\omega_*, z)] \ . \tag{3.2}$$

For lf oscillations, the frequency of which is significantly less than the plasma frequency, the following must be used for ω_* and $\varepsilon(\omega, z)$:

$$\omega_*^2 = [1 - J_0^2(\mathbf{k}_\parallel \mathbf{r}_E)] \frac{\omega_{Li}^2(a)}{1 + \varepsilon_0}$$

$$\varepsilon(\omega, z) = 1 - [1 - J_0^2(\mathbf{k}_\parallel \mathbf{r}_E)] \frac{\omega_{Li}^2(z)}{\omega^2} \tag{3.3}$$

while for hf oscillations they have the form

$$\omega_*^2 = \frac{\omega_{Le}^2(a)}{1 + \varepsilon_0}, \qquad \varepsilon(\omega, z) = 1 - \frac{\omega_{Le}^2(z)}{\omega^2} \ . \tag{3.4}$$

To an accuracy of second order in the mass ratio of electrons to ions, the hf field does not change the spectrum of the hf surface oscillations. Hence, the second and third terms in the curly brackets of Eq. (3.1) and the damping factor [Eq. (3.2)] may be evaluated using the dispersion relation obtained by Stepanov [3], and the last term of Eq. (3.1) corresponds to the correction considered by Romanov [2] for plasma inhomogeneities in the absence of an hf field.

We note that the choice of the point a bounding the transition layer is based only on condition (2.1). For this, the expression for the frequency is independent of the choice of a in the region of slow density change ($a \ll L$) since everywhere in this region $\varepsilon(\omega_*, z) \approx -\varepsilon_0$.

4. As the frequency ω_0 of the external field is decreased and its harmonic $n\omega_0$ approaches ω_* [Eq. (3.4)] a parametric resonance is excited which leads to a growth rate γ for lf (with frequency ω) and hf (with frequencies $\omega \pm n\omega_0$) surface oscillations. From Eq. (2.2) for this case we obtain

$$(\omega + i\gamma)^2 - \frac{\omega_{Li}^2(a)}{1 + \varepsilon_0} J_n^2(\mathbf{k}_\parallel \mathbf{r}_E) \frac{n\omega_0 \Delta}{\Delta^2 - (\omega + i\gamma + i\gamma')^2} = 0. \tag{4.1}$$

Here $\Delta = n\omega_0 - \Omega$, where Ω is defined by Eqs. (3.1) and (3.4).

The dispersion relations 4.1 differ from those of [1] due to inclusion of small corrections to ω_*, which determine the spatial dispersion of the hf surface waves, and also due to inclusion of damping of surface oscillations. Such a difference may be important, as is shown in [4] for the case of a parametric resonance in an unbounded plasma.

We first consider the case of disturbances $n\omega_0 - \omega_*$, for which, in the region of maximum damping coefficient, the condition $\gamma \gg \gamma'$ holds. In this case we obtain from Eq. (4.1) the following expression for the spectrum of periodic instabilities

$$\omega^2 = \frac{1}{4}\left\{\Delta^2 + 2\left[J_n^2(k_\| r_E)\, n\omega_0\Delta \frac{\omega_{Li}^2(a)}{1+\varepsilon_0}\right]^{1/2}\right\} \tag{4.2}$$

$$\gamma = \frac{1}{2}\left\{-\Delta^2 + 2\left[J_n^2(k_\| r_E)\, n\omega_0\Delta \frac{\omega_{Li}^2(a)}{1+\varepsilon_0}\right]^{1/2}\right\}^{1/2} \tag{4.3}$$

$$\Delta > 0$$

and for the aperiodic ($\omega \equiv 0$) instabilities

$$\gamma = \left\{-\frac{\Delta^2}{2} + \left[\frac{\Delta^4}{4} - J_n^2(k_\| r_E)\, n\omega_0\Delta \frac{\omega_{Li}^2(a)}{1+\varepsilon_0}\right]^{1/2}\right\}^{1/2} \tag{4.4}$$

$$\Delta < 0\,.$$

From Eq. (4.3) it follows that the maximum value of the damping coefficient for the periodic instability found in [1],

$$\gamma_{\max} = \left\{(\max J_n^2)\frac{\sqrt{27}}{32}\frac{n\omega_0\omega_{Li}^2(a)}{1+\varepsilon_0}\right\}^{1/4} \tag{4.5}$$

is attained near $\Delta = 2\gamma_{\max}/\sqrt{3}$. Considering that the surface oscillations in this case of a cold plasma have spatial dispersion, we conclude that the maximum in Eq. (4.5), as found in [1], is valid over a relatively wide range of frequencies of the external field consistent with $\gamma \gg \gamma'$. For the case of an aperiodic instability the maximum value of the damping coefficient,

$$\gamma_{\max} = \left[(\max J_n^2)\frac{n\omega_0\omega_{Li}^2(a)}{2(1+\varepsilon_0)}\right]^{1/3} \tag{4.6}$$

is obtained for $\Delta = -\gamma_{\max}$.

We note that the maximum value of the damping coefficient for an aperiodic instability [Eq. (4.6)] exceeds that for a periodic one [Eq. (4.5)]. Consequently, an aperiodic instability may completely define the nonlinear parametric interaction even for frequencies of the external field $\omega_0 > \omega_*/n$.

For given density profiles in the transition region (e.g., a linear distribution) damping of surface waves $\gamma' \sim \omega_p k_\| a$ and simultaneous satisfaction of the conditions $\gamma \gg \gamma'$ and $k_\| r_E \sim 1$ is possible only for sufficiently large external fields, i.e., $(r_E/a) \ll (\omega_0/\gamma_{\max})$.

For values of $n\omega_0 - \omega_*$ such that $\omega, \gamma \ll \gamma'$, the solution of the dispersion relation (4.1) for a periodic instability has the form

$$\omega^2 = J_n^2(k_\| r_E)\frac{\omega_{Li}^2(a)}{1+\varepsilon_0}\frac{n\omega_0\Delta}{\Delta^2+(\gamma')^2}, \qquad \gamma = J_n^2(k_\| r_E)\frac{\omega_{Li}^2(a)}{1+\varepsilon_0}\frac{n\omega_0\Delta\gamma'}{[\Delta^2+(\gamma')^2]^2} \tag{4.7}$$

$$\Delta > 0\,.$$

The corresponding damping coefficient for an aperiodic instability is

$$\gamma = \left[-J_n^2(k_\| r_E)\frac{\omega_{Li}^2(a)}{1+\varepsilon_0}\frac{n\omega_0\Delta}{\Delta^2+(\gamma')^2}\right]^{1/2}\,. \tag{4.8}$$

It should be noted that thermal motion has an insignificant effect on the spectrum of the above oscillations when the characteristic size of the inhomogeneity in the transition layer is large compared to the Debye length. The opposite limiting case of an inhomogeneous plasma with a sharp boundary is discussed in [5].

We note that the preceding discussion dealt with the case of surface waves of wavelength $(1/k_\|)$ much less than the thickness of the dielectric layers d, bounding the plasma. For an arbitrary ratio of these lengths, it is necessary to substitute in the above equations the following:

$$\varepsilon_0 \to \varepsilon_0 \frac{(1+\varepsilon_0)\exp(k_\parallel d) + (1-\varepsilon_0)\exp(-k_\parallel d)}{(1+\varepsilon_0)\exp(k_\parallel d) - (1-\varepsilon_0)\exp(-k_\parallel d)} .$$

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